Do Dogs Know Related Rates Rather than Optimization?

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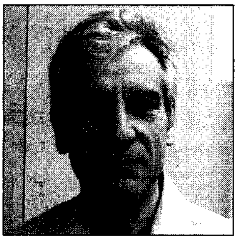

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Timothy J. Pennings (this *Journal*, [**2**]) describes the strategy followed by his dog, Elvis, to fetch a tennis ball thrown into the water from the shore of Lake Michigan.

Let AC in Figure 1 be the water's edge. The ball is thrown from A, and falls into the water at B. Elvis, said Pennings, did not jump immediately in the water at A, a strategy that would have minimized the distance traveled (AB). Neither did he run along the beach to enter at C, which would have minimized the swimming distance (BC). Rather, he ran along the beach a part of the way, then jumped into the water at a point D, somewhere between A and C. Pennings speculated that the location of D was chosen to minimize the retrieval time. In order to test his hypothesis, he measured the running speed (r) and the swimming speed (s) of his dog (r being considerably larger than s) and computed the optimal path.

The time to get the ball is given by

$$T(y) = \frac{z-y}{r} + \frac{\sqrt{x^2+y^2}}{s}. \tag{1}$$

The value of y providing the optimal path is the value for which $T'(y) = 0$. Solving $T''(y) = 0$ for y, Pennings obtained:

$$y = \frac{x}{\sqrt{r/s+1}\sqrt{r/s-1}} \tag{2}$$

Surprisingly, it turned out that in most cases Elvis jumped into the water at a point that agreed remarkably well with the optimal value given by the mathematical model. Pennings did *not* conclude that his dog knows calculus, but instead noted that "Elvis's

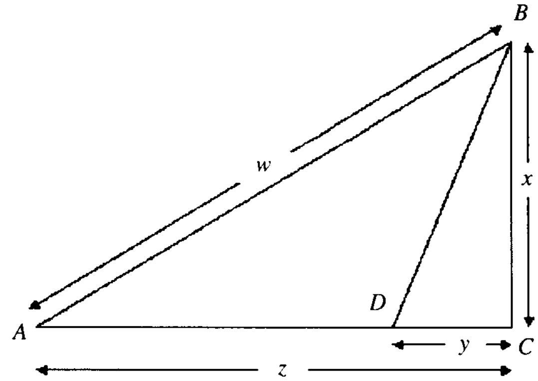

Figure 1. The problem space (adapted from Pennings)

behavior is an example of the uncanny way in which nature (or Nature) often finds optimal solutions" (p. 182).

In an effort to double the sample size, we determined that Salsa, a female Labrador, also apparently chooses the optimal path when playing fetch along a lakeside beach near Nimes in France! But we want to suggest that the dog's behavior may not be as uncanny as it might at first seem. What makes the dog's performance seem surprising is that it agrees with the result of a mathematical model minimizing *the total duration of the travel.* That is, it suggests that dogs are supposedly able to calculate optimal strategies involving knowledge of the entire route before they ever begin running. But the question is; is this ability really required?

Let us assume instead that the dogs are attempting to optimize their behavior *on a moment-to-moment basis.* For our specific concern, let us assume that a dog playing fetch chooses at each point in time the path that allows it to maximize its speed of approach to the ball. When running from A towards C at a constant speed, the ball at B appears closer and closer as the dog gets closer to C, but its speed of approach to B diminishes (reaching zero at C). At some moment of his run, his speed of approach while running on the beach becomes equal to his speed of approach when swimming directly to the ball; i.e., his swimming speed. It can be shown (a related rates problem) that if the dog jumps into the water at this moment, this strategy yields the same y value as that provided by the time of travel minimization model.

For let $W(t)$ be the distance from the dog to the ball (see Figure 1). Then

$$W(t) = \sqrt{x^2(t) + z^2(t)},$$

so

$$W'(t) = \frac{xx' + zz'}{\sqrt{x^2 + z^2}}.$$

Since $x' = 0$ and $z' = r$, we get

$$W'(t) = \frac{rz}{\sqrt{x^2 + z^2}}.$$

When $z = y$, $W'(t) = s$, so we solve

$$s = \frac{ry}{\sqrt{x^2 + z^2}}$$

for y to get

$$y = \frac{x}{\sqrt{r/s + 1}\sqrt{r/s - 1}}, \tag{3}$$

which is identical to (2).

Although this solution is identical to that proposed by Pennings, it was gained without assuming canine knowledge of the entire route, and hence can be construed as a more plausible model for dog's strategy. To perform in this way, dogs must first be able to estimate accurately their speed of approach at each moment. Second, they need to have a general awareness of their swimming speed before entering the water, since they have to jump into the water at the point when their speed of approach towards the ball while running on the beach becomes slower than their swimming speed.

Is it reasonable to postulate that dogs have this ability? To answer, we ask in turn: Would this ability have been useful for dog's ancestors, who lived in a natural environment? Obviously, the ability to detect transient changes in distance is crucial for animal species' survival, as when the ball is replaced by prey or predator. The general awareness of swimming speed is certainly a more sophisticated ability, in so far as it requires memory of relative speeds. But again, it seems to be essential for the survival of any animal to know how rapidly it can move in the various media that it may encounter when pursuing prey or escaping a predator. For animal species, such as mammals, that are destined to move in a variety of different mediums, it is reasonable to assume that there is an innate ability to learn quickly from early experiences.

In conclusion, both approaches require the use of calculus, either in solving an optimization problem or a related rates problem. However, as we showed, there is a major difference between our interpretation and the interpretation that results from taking Pennings' model as a realistic model of the dog's strategy. The ability that is required, in our view, forms part of general motion detection capabilities. As motion detection is common in most animals, it has been the focus of thorough investigations that have revealed some of its biological mechanisms (for a review, see **[1]**). Thus our solution provides a bridge between a specific behavioral strategy and ubiquitous biological mechanisms. Calculus then allows us to demonstrate that evolution has led to the development of biological mechanisms that are so powerful that they often lead to the optimal solution.

References

1. C.W.G. Clifford and M.R. Ibbotson, Fundamental mechanisms of visual motion detection: models, cells and functions, *Progress in Neurobiology* **68** (2003) 409–437.
2. T.J. Pennings, Do dogs know calculus? *College Math. J.* **34** (2003) 178–182.

Since his appearance on the cover of this journal in 2003, Elvis has gone on to greater things, including being awarded an honorary degree "Litterarum Doctoris Caninarum" from Hope College in January, 2005 (see photograph). He was also the subject of a chapter in the book *The Math Instinct* by Keith Devlin (Thunder's Mouth Press, 2005).

The author of the 2003 article "Do Dogs Know Calculus?", Tim Pennings, (pennings@hope.edu), reports that his research with Elvis continues. One of his more recent findings is that, contrary to the expectations expressed in the article "Do Dogs Know Related Rates Rather Than Optimization?", Elvis appears to make global decisions rather than just instantaneous decisions when retrieving the ball.

Pennings writes, "I discovered this by accident. Playing fetch with Elvis, I decided to throw the stick while standing in the water, about 10-12 feet from shore, and with Elvis right beside me. When I threw the stick in a path parallel to the beach, Elvis swam in to shore, ran along the beach for a sizeable distance, and then dove back into the water to retrieve the stick. Thus, in swimming to shore he was not acting to minimize his distance to the stick as quickly as possible. Instead, he DID in fact apparently make a 'global' decision from the outset as to what path would get him to the stick the most quickly."